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UPPER CRITICAL FIELD ON THE BASIS OF A TWO-BAND MODEL

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The discovery of the high temperature of transition in superconducting state $T_c \sim 40\text{K}$ in the simple intermetallic compound MgB_2 [1] has stimulated researches of these material properties both in the experimental and theoretical plans. The significant result of these researches is the discovery of two energetic gaps in the spectrum of the elementary excitations [2] and the possibility of theoretical describing of this compound on the base of the two-band model [3].

This two-band model and its generalization for the anisotropic value of the energetic gaps D_1 and D_2 case, [4, 5] confirm the experimental results for the thermal capacity C_s dependence on temperature, the penetration depth of the magnetic field and other characteristics in the MgB_2 compound.

As it is known, the superconductivity metals undergo a transition from the superconductivity phase into normal in the magnetic field at some its value. This transition which is relevant to the full penetration of the magnetic field into the superconductor, occurs in the moment when the field achieves the value of the upper critical field H_{c2} .

The main purpose of the work is researching of pure two-band superconductor of the secondary type for arbitrary temperatures close to the upper critical field and the definition of temperature dependence of the H_{c2} value. The calculations are based on the fundamental equations of the electrodynamics of two-band superconductors [6], which are valid both for pure and doped superconductors. If the external magnetic field is great enough the order parameters D_m ($m = 1, 2$) of two-band superconductor is small, and we can use equations ref. [6] for pure two-band superconductor:

$$\Delta_m^{\square}(\vec{x}) = \frac{1}{b} \sum_w \sum_{nn'} V_{nm} \int d\vec{y} g_{n'n}^{\rightarrow}(\vec{y}, \vec{x}/w) \Delta_{n'}^{\rightarrow}(\vec{y}) g_{n'n}^{\rightarrow}(\vec{y}, \vec{x}/-w). \quad (1)$$

We restricted here by linear terms on Δ_n quantities in comparison with reduced in [6] because in the $H = H_{c2}$ point occurs solutions with the infinitely small values Δ_m . Green function defines by equation at presence of the magnetic field [7]:

$$g_{nn'}(r, r'/w) = e^{ij(r, r')} g_{n'n}^0(r, r'/w),$$

$$j(r, r\odot) = e \int_{r\odot}^r A(\vec{l}) d\vec{l}, \quad (2)$$

where $g_{n'n}^0$ - Green function of an electron in normal metal without magnetic field. The presence of the magnetic field is taken into account by the phase multiplier.

We decompose in equation (1) the normal metal function $g_{n'n}^0$ into the row by the Bloch functions $\Psi_{nk}^{\rightarrow}(x) = e^{i(\vec{k}x)} U_{nk}^{\rightarrow}(x)/\sqrt{N}$ (U_{nk}^{\rightarrow} - the Bloch amplitude):

$$g_{n\otimes n}^0\left(\begin{matrix} \rightarrow \\ y, x/w \end{matrix}\right) = \sum_{\substack{\rightarrow \\ k \otimes k \otimes}} g_{n\otimes n}^0\left(\begin{matrix} \rightarrow \\ k \otimes, k/w \end{matrix}\right) \Psi_{n\otimes k \otimes}^{\rightarrow}\left(\begin{matrix} \rightarrow \\ y \end{matrix}\right) \Psi_{nk}^*\left(\begin{matrix} \rightarrow \\ x \end{matrix}\right). \quad (3)$$

and use approaching of the diagonal Green functions. Magnetic field is guided along the z-axis. Besides choose the vector potential as $A_x = A_z = 0$; $A_y = H_0 x$.

On the base of (1) - (3) after calculating the integral by impulse \vec{k} , averaging by elementary cells, applying Maki and Tsuzuki methods [7] and assuming $\Delta_m(x) = \Delta_m \exp\{H_0 x^2\}$, we obtain:

$$\Delta_m^* = \sum_n V_{nm} N_n \rho_n^{-1/2} \Delta_n^* \int_1^{\infty} \frac{du}{u} \int_{\delta_n'}^{\infty} d\zeta \exp\left[-\frac{\zeta^2}{4}(u^2+1)\right] \frac{I_0\left[\frac{\zeta^2}{4}(u^2-1)\right]}{\text{sh}\left[u\zeta\rho_n^{-1/2}\right]}, \quad (4)$$

where n_n and N_n are accordingly electron speed and electron density of state on n-th cavity of the Fermi surface,

$$d_n' = (eH_0)^{1/2} \frac{n_n}{2ge_0 w_D^{(n)}},$$

$$r_n = \frac{v_n^2 eH_0}{(2pT)^2} \quad (5)$$

where γ is the Euler constant, $w_D^{(n)}$ is the cut-off frequency.

Basing on (5) it is easy to obtain the equation for the upper critical field definition in the next view

$$-\Delta_m^2 + \sum_n V_{nm} N_n \Delta_n^2 \ln \frac{2\gamma w_D^{(n)}}{\pi T_c} + \sum_n V_{nm} N_n \Delta_n^2 \left[\ln \frac{T_c}{T} - f(\rho_n) \right] = 0, \quad (6)$$

where

$$f(r_n) = r_n^{-1/2} \int_0^{\infty} dz \int_0^{\infty} \frac{du}{u \text{sh}[uz r_n^{-1/2}]} \times \left\{ 1 - \exp\left[-\frac{z^2}{4}(u^2+1)\right] I_0\left[\frac{z^2}{4}(u^2-1)\right] \right\} \quad (7)$$

Equality to zero of the system (6) determinant corresponds to the presence of non-zero solutions, that is the connected pairs forming. The field, in the presence of which such solutions can appear, is the upper critical field H_{C2} . So the H_{C2} value defines from the condition of the system (6) solvability:

$$a f(r_1) f(r_2) + B_1 f(r_1) + B_2 f(r_2) + C = 0, \quad (8)$$

where

$$B_n = N_n V_{nm} - \alpha \xi_c^{(n)} \quad (n = 1, 2).$$

$$C = 1 - N_1 V_{11} x_T^{(1)} - N_2 V_{22} x_T^{(2)} + a x_T^{(1)} x_T^{(2)}; \quad a = N_1 N_2 (V_{11} V_{22} - V_{12} V_{21});$$

$$x_T^{(n)} = \ln \frac{2g w_D^{(n)}}{pT}; \quad x_c^{(n)} = \ln \frac{2g w_D^{(n)}}{pT_c}.$$

The analytic solutions to equation (8) could be computed for two limit cases as follows:

$$a. \rho_n \ll 1 \quad (T_c - T \ll T_c); \quad b. \rho_n \gg 1 \quad (T \ll T_c).$$

For which functions $f(r_n)$ are defined in works [7]:

$$f(r_n) = \frac{7}{6}z(3)r_n - \frac{31}{10}z(5)r_n^2 + \frac{381}{28}z(7)r_n^3, \quad r_n \ll 1 \quad (9)$$

$$f(r_n) = \ln \frac{2(2g r_n)^{1/2}}{e_0} - \frac{1}{p^2 r_n} \left[z'(2) + \frac{z(2)}{2} \ln \frac{2}{p^2 g r_n} \right], \quad r_n \gg 1. \quad (10)$$

In the case of $(T_c - T \ll T_c)$ applying the formulas in (10) and (11) we obtain the following expression for the H_{c2} value .

$$H_{c2}(T) = \frac{4p^2 T_c^2}{e} [v_1^2 h_1 + v_2^2 h_2]^{-1} \frac{6}{7x(3)} \Theta \cdot \left[1 + \Theta \left\{ \frac{\frac{v_1^2 h_1 + v_2^2 h_2}{v_2^2 h_1 + v_1^2 h_2} \frac{31}{10} x(5) \left(\frac{6}{7x(3)} \right)^2 \frac{3}{2}}{\left(\frac{v_1 h_1 + v_2 h_2}{v_2 h_1 + v_1 h_2} \right)} \right\} \right], \quad (11)$$

$$q = 1 - \frac{T}{T_c}, \quad h_1 = \frac{1}{2}(1+h), \quad h_2 = \frac{1}{2}(1-h);$$

$$h = \frac{N_1 V_{11} - N_2 V_{22}}{\sqrt{(N_1 V_{11} - N_2 V_{22})^2 + 4N_1 N_2 V_{12} V_{21}}}. \quad (12)$$

In the b) case (T close to zero) we obtain:

$$\frac{H_{c2}(T)}{H_{c2}(0)} = \left[1 + \frac{16\gamma}{e_0^2 \pi^2} \left(\frac{T}{T_c} \right)^2 e^{Sv(\lambda) - Sv(1)} \left\{ \left(\lambda \gamma^+ + \frac{1}{\lambda} \gamma^- \right) + \left[\zeta'(2) + \zeta(2) \ln \frac{4T}{e_0 \pi T_c} + \frac{S}{2} n(1) - \frac{S}{2} n(1) \right] + \frac{z(2)}{2} \left(I g^+ - \frac{1}{I} g^- \right) \ln I \right\} \right], \quad (13)$$

where

$$n(I) = \sqrt{(\ln I - h^{(-)})^2 + \frac{4N_1 N_2 V_{12} V_{21}}{a^2}}, \quad S = \pm 1$$

$$g^\pm = \frac{1}{2} \left[1 \pm \frac{h^- - \ln I}{Sv(I)} \right], \quad H_{c2}(0) = \frac{p^2 T_c^2 e_0^2}{2g e v_1 v_2} \exp[n(1) - n(I)]. \quad (14)$$

We obtained equation (8), on the base of which the value of the upper critical field in the two-band model can be calculated on the whole temperature interval $0 \leq T \leq T_c$. The analytic solutions of this equation were obtained for $T \approx T_c$ (11) and $T \approx 0$ (13). It is easy to notice that H_{c2} depends on the correlations of the speeds n_I

and n_2 of the electrons on the Fermi surface, and on the constants of the electronic-phonon interaction I_{nm} .

If $H_{c_2}^0(0)$ and T_{c0} are introduced (upper critical field and critical temperature of the one-band low-temperature superconductor), on the base of (14) we obtain:

$$H_{c_2}(0)/H_{c_2}^0(0) = (T_c/T_{c0})^2 \frac{v_1}{v_2} \exp(v(1) - v(I)). \quad (15)$$

The numerical estimations let us come to conclusion, that the upper critical field of two-band superconductors for $T=0$ can exceed the value of $H_{c_2}^0(0)$ for usual superconductors by two-three orders. These big values $H_{c_2}(0)$ are provided by high T_c and by ratio $v_1/v_2 > 1$ or $\gg 1$.

Our goal is to research the dependence from the temperature of the value H_{c_2} for the connection MgB_2 in the whole temperature interval $0 < T < T_c$. For this is necessary to estimate the parameters of the two band theory I_{nm} , using the experimental data obtained by researching this substance [8], [9]:

$$\Delta_1(0)=6,8\text{mev}; Z(0)=\frac{\Delta_1(0)}{\Delta_2(0)}=3,8, \frac{N_1}{N_2}=0,8, \left. \frac{C_S - C_N}{C_N} \right|_{T=T_c} = 0,78. \quad (16)$$

In correspondence with the two-band theory of superconductors we have [10]

$$\Delta_1(0)=6,8\text{mev}; Z(0)=\frac{\Delta_1(0)}{\Delta_2(0)}=3,8, \frac{N_1}{N_2}=0,8, \left. \frac{C_S - C_N}{C_N} \right|_{T=T_c} = 0,78, \quad (17)$$

where

$$V(0) = \frac{N_2 V_{22} - N_2 V_{12} / Z(0)}{a},$$

$$a \ln Z(0) = N_1 V_{11} - N_2 V_{22} + \frac{V_{12} N_2}{Z(0)} - N_1 V_{12} Z(0). \quad (18)$$

Let's consider that $N_2 V_{22} = 0$, which corresponds to the passivity of the second zone in forming of the Cooper pairs. On the base of data (16) and equations (18) we get: for $w_D^{(1)} = w_D^{(2)} = 700 \text{ K}$, $N_1 V_{11} = 0,3$, $N_2 V_{12} = 0,12$; for $w_D^{(1)} = 500 \text{ K}$, $N_1 V_{11} = 0,33$, $N_2 V_{12} = 0,13$; for $w_D = 300 \text{ K}$, $N_1 V_{11} = 0,42$, $N_2 V_{12} = 0,16$.

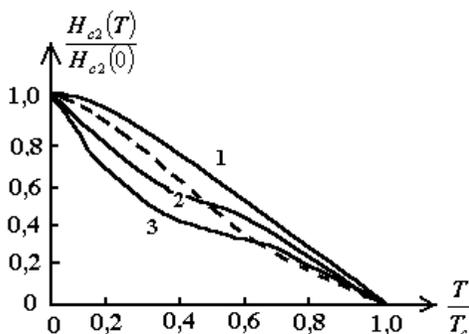
The heat capacity jump at $T = T_c$ is determined by the relation [10]

$$\frac{C_S - C_N}{C_N} = 1,43 \frac{(N_1 + N_2 / Z_c^2)^2}{(N_1 + N_2)(N_1 + N_2 / Z_c^4)},$$

$$Z_c = \frac{\Delta_1(T \rightarrow T_c)}{\Delta_2(T \rightarrow T_c)} = \frac{1 - N_2 V_{22} x_c}{N_1 V_{12} x_c}. \quad (19)$$

The estimate of the ratio Z_c on the base of experimental data (16) and on the base of the first formula (19) gives the value $Z_c = 3,19$. The second formula (19) gives the value $Z_c = 3,26$. So we conclude that $D_1(T)/D_2(T)$ isn't a constant value since $Z(0) > Z_c$.

On the fig.1 is presented the dependence of the upper critical field H_{c2} from the temperature, which is obtained by numerical method of solving the equation (8) and using its analytic solutions when $T \ll T_c$ (13) and $T \sim T_c$ (11).



The temperature dependence $H_{c2}(T)/H_{c2}(0)$ at $N_1V_{11} = 0,3$, $N_2V_{12} = 0,12$, $N_1/N_2 = 0,8$ and $v_1/v_2 = 1$, $v_1/v_2 = 2$, $v_1/v_2 = 3$ (curves 1,2,3 correspondingly), curve interrupted corresponds to the experimental dependence [11].

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